

Direct Adaptive Control Based on Gradient Estimation*

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Abstract A control method of direct adaptive control based on gradient estimation is proposed in this article. The dynamic system is embedded in a linear model set. Based on the embedding property of the dynamic system, an adaptive optimal control algorithm is proposed. The robust convergence of the proposed control algorithm has been proved and the static control error with the proposed method is also analyzed. The application results of the proposed method to the industrial polypropylene process have verified its feasibility and effectiveness.

Keywords direct adaptive control, embedding property, gradient estimation, robust convergence, static error, polypropylene process

1 INTRODUCTION

In the last 30 years, model predictive control has been developed to fill the large gap between the state of the art in control theory and its actual applications. The vast majority of predictive control applications are based on linear dynamic models generated at some particular operating points. Though, linear predictive control can be extended to effectively handle a variety of simple nonlinear dynamic systems, there are cases where nonlinear effects are significant enough to justify the linear predictive control may not be adequate [1]. These include at least two broad categories: (1) Regulator control problems where the processes are highly nonlinear and subject to large frequent disturbances (polymer manufacturing, *etc.*); (2) Servo control problems where the operating points change frequently and span a sufficient wide range of nonlinear process dynamics (pH control, ammonia synthesis, *etc.*).

For model predictive control methods, the control system has to be designed with high-level expertise that mass users do not have. Due to the complexity of implementing these methods, commercially practical control of complex systems is difficult and expensive. Consequently, a need exists for a generic controller that can be used easily and effectively to control a wide variety of simple and complex systems. The controller should have good self-learning and adaptive capabilities to cope with uncertainties and nonlinearity in the system.

Initial progress was made a few years ago in the effort to tackle the problem of model-free adaptive control. For instance, an identification-free algorithm was discussed by Marsik and Strejic for adaptive control of industrial processes [2]. Performance criteria based on geometrical properties of the process control error is used to automatically adjust several parameters of the controller while the adaptation acts like an ordinary control loop. Sine-wave signals is needed in the justification of a heuristic proposition based on which the controller tuning algorithm is needed. Similar principle is followed in the design of an auto-tuner, a name given by Astrom for his knowledge-

based intelligent Proportion-Integral-Differential (PID) controller [3]. A simple characterization of the non-oscillatory process dynamics from the step response, such as the normalized dead time and the normalized process gain, is used to guide the PID control based on Ziegler-Nichols tuning.

In recent years, neural networks (NN) have shown great potential to be applied to the control and optimization problems, because of the fact that NN can approximate a wide range of nonlinear functions to any desired degree of accuracy under certain conditions. It was shown that for stable and efficient on-line control using the backpropagation (BP) learning algorithm, the identification must be sufficiently accurate before control action is initiated [4–6]. Several good NN control approaches have been proposed based on Lyapunov's stability theory [7–10]. One main advantage of these schemes is that the adaptive laws are derived based on Lyapunov synthesis method and therefore guarantee the stability of continuous-time systems without the requirement of off-line training. However, all these elegant methods in continuous-time domain are not directly applicable to discrete-time systems [11].

Spall and Cristion have also made contributions toward the generic controller design [12, 13]. In their methods, the NN is used to directly model the resulting unknown control law without the need to construct a separate model for the unknown process dynamics. Due to the feed back of the unknown process dynamics, stochastic approximation algorithm is used to estimate the connection weights in the NN instead of the standard gradient-descent-type algorithm. In this article, a method of direct adaptive control based on gradient estimation is proposed.

2 DIRECT ADAPTIVE CONTROL BASED ON GRADIENT ESTIMATION

2.1 Embedding property

For convenience, the system is considered

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one-step lag dynamic discrete system, define $F: \{S_i, i \in I\}$ as the dynamic system set, where S_i is the dynamic system, I is the index set. If for dynamic system S , there is

$$\mathbf{y}(k) = S[\mathbf{u}(k-1), \mathbf{Y}_{k-1}, \mathbf{U}_{k-2}] \quad (1)$$

then $[\mathbf{u}(k-1), \mathbf{y}(k)]$ is called a data set for dynamic system S , where

$$\mathbf{Y}_{k-1} = [y_1(k-1), \dots, y_1(k-N_y), \dots, y_n(k-1), \dots, y_n(k-N_y)]^T \in \mathbf{R}^{nN_y} \quad (2)$$

$$\mathbf{U}_{k-2} = [u_1(k-2), \dots, u_1(k-N_u-1), \dots, u_m(k-1), \dots, u_m(k-N_u-1)]^T \in \mathbf{R}^{mN_u} \quad (3)$$

Definition 1: If for any data set $[\mathbf{u}(k-1), \mathbf{y}(k)]$ of S , there exists a dynamic system S_i^k , $S_i^k \in F$ satisfies

$$\mathbf{y}(k) = S_i^k[\mathbf{u}(k-1), \mathbf{Y}_{k-1}, \mathbf{U}_{k-2}] \quad (4)$$

then S is called that it can be embedded in the set $F: \{S_i, i \in I\}$.

Theorem 1: For dynamic system S , if $\mathbf{u}(k-2) = \mathbf{u}(k-1)$ leads to $\mathbf{y}(k-1) = \mathbf{y}(k-2)$, where $\mathbf{y}(k-1) = S[\mathbf{u}(k-2), \mathbf{Y}_{k-2}, \mathbf{U}_{k-3}]$, $\mathbf{y}(k) = S[\mathbf{u}(k-1), \mathbf{Y}_{k-1}, \mathbf{U}_{k-2}]$ then there always exists an embedding matrix $\mathbf{G}(k)$, satisfies

$$\mathbf{y}(k) - \mathbf{y}(k-1) = \mathbf{G}(k)[\mathbf{u}(k-1) - \mathbf{u}(k-2)] \quad (5)$$

Proof sees Appendix A.

Corollary 1: For multi-input single output (MISO) system, if $\mathbf{u}(k-2) = \mathbf{u}(k-1)$, where $\mathbf{u} \in \mathbf{R}^m$ leads to $\mathbf{y}(k-1) = \mathbf{y}(k)$, where $\mathbf{y} \in \mathbf{R}$, then there exists an embedding vector $\mathbf{g}(k)$ satisfies,

$$\mathbf{y}(k) - \mathbf{y}(k-1) = \mathbf{g}^T(k)[\mathbf{u}(k-1) - \mathbf{u}(k-2)] \quad (6)$$

Similar to **Theorem 1**, **Corollary 1** can be proved. Obviously, $\mathbf{g}(k)$ is not single, but there is the following **Corollary 2**.

Corollary 2: If $\mathbf{g}(k)$ is an arbitrary vector, satisfies Eq. (6) and

$$\mathbf{g}(k) = \frac{\mathbf{u}(k-1) - \mathbf{u}(k-2)}{\|\mathbf{u}(k-1) - \mathbf{u}(k-2)\|^2} \{\mathbf{y}(k) - \mathbf{y}(k-1)\} \quad (7)$$

where, $\mathbf{u}(k-1) - \mathbf{u}(k-2) \neq 0$, $\|\cdot\|$ is the Euclid norm, then

$$\|\mathbf{g}(k)\|^2 = \min_{\mathbf{g}} \|\mathbf{g}\|^2 \quad (8)$$

Proof sees Appendix B.

Obviously, the embedding matrix $\mathbf{G}(k)$ depends on $\mathbf{y}(k)$, $\mathbf{y}(k-1)$, $\mathbf{u}(k-1)$, $\mathbf{u}(k-2)$, define $\mathbf{F}_G = \{\mathbf{G}(k, \mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_1, \mathbf{u}_2); \mathbf{y}_j \in \mathbf{Y}_k, \mathbf{u}_j \in \mathbf{U}_{k-1}; j=1, 2; k=0, 1, 2, \dots\}$, or any element of $\mathbf{G}(k) = \mathbf{G}(k, \mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_1, \mathbf{u}_2) \in \mathbf{F}_G$, there is a linear system that satisfies

$$\mathbf{y}(k) - \mathbf{y}(k-1) = \mathbf{G}(k)[\mathbf{u}(k-1) - \mathbf{u}(k-2)] \quad (9)$$

where, $\mathbf{y}(k) = \mathbf{y}_1$, $\mathbf{y}(k-1) = \mathbf{y}_2$, $\mathbf{u}(k-1) = \mathbf{u}_1$,

$$\mathbf{u}(k-2) = \mathbf{u}_2.$$

Corollary 3: The dynamic system S is assumed to be one-step lag dynamic discrete system, where its input $\mathbf{u}(k-1) \in \mathbf{U}_{k-1}$, output $\mathbf{y}(k) \in \mathbf{Y}_k$, then S can be embedded in the dynamic system set $\mathbf{F}_G = \{\mathbf{G}(k, \mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_1, \mathbf{u}_2); \mathbf{y}_j \in \mathbf{Y}_k, \mathbf{u}_j \in \mathbf{U}_{k-1}; j=1, 2; k=0, 1, 2, \dots\}$ and satisfies Eq. (9), where,

$$\mathbf{G}(k, \mathbf{y}_1, \mathbf{y}_2, \mathbf{u}_1, \mathbf{u}_2) = \begin{cases} \mathbf{G}_0, & \mathbf{u}_1 = \mathbf{u}_2 \\ \frac{[\mathbf{y}(k) - \mathbf{y}(k-1)][\mathbf{u}(k-1) - \mathbf{u}(k-2)]^T}{\|\mathbf{u}(k-1) - \mathbf{u}(k-2)\|^2}, & \mathbf{u}_1 \neq \mathbf{u}_2 \end{cases} \quad (10)$$

For multi-step lag system, for instance two-step lag system, similar to **Theorem 1**, the embedding property can be constructed as

$$\mathbf{y}(k) - \mathbf{y}(k-2) = \mathbf{G}(k)[\mathbf{u}(k-1) - \mathbf{u}(k-2)] + \mathbf{G}(k-1)[\mathbf{u}(k-2) - \mathbf{u}(k-3)] \quad (11)$$

where, $\mathbf{G}(d), d=k, k=-1$ satisfy

$$\mathbf{G}(d) = \begin{cases} \mathbf{G}_0, & \mathbf{u}(d-1) = \mathbf{u}(d-2) \\ \frac{[\mathbf{y}(d) - \mathbf{y}(d-1)][\mathbf{u}(d-1) - \mathbf{u}(d-2)]^T}{\|\mathbf{u}(d-1) - \mathbf{u}(d-2)\|^2}, & \mathbf{u}(d-1) \neq \mathbf{u}(d-2) \end{cases} \quad (12)$$

To build a model-based controller for nonlinear dynamic system, the first step is to construct a model which is convenient for controller design. The embedding property can be looked as a kind of gain scheduling [14], where the nonlinear dynamic system is described by the linear parameter-varying model. Therefore, linear controller design methods could be applied to design the linear controller for the linear parameter-varying model. Based on the embedding property, the direct adaptive controller for dynamic system S can be designed.

2.2 Direct adaptive controller based on gradient estimation

To design the controller for the linear parameter-varying model, the following problems need to be solved. The first is how to build the structure of the controller. The second is how to estimate the time-varying model parameter and update the controller. The last is how to deal with the control input and output constraints.

Without losing generality, the controlled system is assumed to have the embedding property as **Corollary 3**. Then, the direct adaptive control can be described as that known as the data set $(\mathbf{Y}_k, \mathbf{U}_{k-1})$ and the optimal control objective $J[\mathbf{u}(k), \mathbf{y}(k+1)]$ at the time instant k , finding the control vector $\mathbf{u}(k)$ to minimize the optimal control objective $J[\mathbf{u}(k), \mathbf{y}(k+1)]$. The optimal control objective is defined as

$$J(\mathbf{u}, \mathbf{y}) = \rho \mathbf{u}^T \mathbf{u} + r \Delta \mathbf{u}^T \Delta \mathbf{u} + (\mathbf{y} - \mathbf{y}_d)^T (\mathbf{y} - \mathbf{y}_d) \rightarrow \min \quad (13)$$

where, $0 \leq \rho < 1, 0 \leq r < 1$ are scalar constants, $\mathbf{y}_d$ is

the control target. According to the embedding property, let $\frac{\partial J}{\partial \mathbf{u}} = 0$ and after some transformations,

$$\mathbf{u}(k) = \mathbf{u}(k-1) - [(\rho+r)\mathbf{I} + \mathbf{G}^T(k+1)\mathbf{G}(k+1)]^{-1} \cdot [\rho\mathbf{u}(k-1) + \mathbf{G}^T(k+1)(\mathbf{y}(k) - \mathbf{y}_d)] \quad (14)$$

Since $\mathbf{u}(k), \mathbf{y}(k+1)$ are unknown, $\mathbf{G}(k+1)$ cannot be calculated directly, therefore there is a need to estimate $\mathbf{G}(k+1)$ before calculating the control action. Assume the controlled system S is continuously differentiable, then

$$\mathbf{y}[\mathbf{u}(k-1) + \Delta\mathbf{u}] = \mathbf{S}[\mathbf{u}(k-1)] + \left. \frac{\partial \mathbf{S}}{\partial \mathbf{u}} \right|_{\mathbf{u}(k-1)} \Delta\mathbf{u} + o(\|\Delta\mathbf{u}\|^2) \quad (15)$$

Neglecting the high-order bias, then

$$\Delta\mathbf{y} = \mathbf{G} \big|_{\mathbf{u}} \Delta\mathbf{u} \quad (16)$$

where, $\mathbf{G} = \frac{\partial \mathbf{S}}{\partial \mathbf{u}}$ is the gradient matrix. Comparing Eq. (9) and Eq. (16), the embedding matrix can be estimated with the gradient approximation. Therefore, the real control action can be calculated with the gradient approximation as

$$\hat{\mathbf{u}}(k) = \mathbf{u}(k-1) - [(\rho+r)\mathbf{I} + \hat{\mathbf{G}}^T(k+1)\hat{\mathbf{G}}(k+1)]^{-1} \cdot [\rho\mathbf{u}(k-1) + \hat{\mathbf{G}}^T(k+1)(\mathbf{y}(k) - \mathbf{y}_d)] \quad (17)$$

When the embedding matrix is replaced by the gradient matrix $\mathbf{G}$, according to Eq. (9),

$$\mathbf{y}(k) = \mathbf{G}\mathbf{u}(k-1) + \mathbf{y}(0) \quad (18)$$

Considering the multi-step lag system, for instance two-step lag system, the real control action as Eq. (17) can be modified as

$$\begin{aligned} \hat{\mathbf{u}}(k) = & \mathbf{u}(k-1) - \\ & [(\rho+r)\mathbf{I} + \hat{\mathbf{G}}^T(k+1)\hat{\mathbf{G}}(k+1)]^{-1} \cdot \\ & [\rho\mathbf{u}(k-1) + \hat{\mathbf{G}}^T(k+1)(\mathbf{y}(k-1) + \\ & \hat{\mathbf{G}}(k)(\mathbf{u}(k-1) - \mathbf{u}(k-2)) - \mathbf{y}_d)] \end{aligned} \quad (19)$$

Considering the input and output constraints, the direct adaptive control can be described as

$$\begin{aligned} \min_{\mathbf{u}} J(\mathbf{u}, \mathbf{y}) = & \rho\mathbf{u}^T\mathbf{u} + r\Delta\mathbf{u}^T\Delta\mathbf{u} + (\mathbf{y} - \mathbf{y}_d)^T(\mathbf{y} - \mathbf{y}_d) \\ & \mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max} \\ \text{s.t. } & |\Delta\mathbf{u}| \leq \Delta\mathbf{u}_{\max} \\ & \mathbf{y}_{\min} \leq \mathbf{y} \leq \mathbf{y}_{\max} \end{aligned} \quad (20)$$

According to the embedding property, the constant term in the objective function is omitted, and some transformations are done, then Eq. (20) as

$$\begin{aligned} \min_{\mathbf{u}} & (\mathbf{u}^T \mathbf{H} \mathbf{u} - \mathbf{d}^T \mathbf{u}) \\ \text{s.t. } & \mathbf{C} \mathbf{u} \leq \mathbf{c} \end{aligned} \quad (21)$$

where, $\mathbf{H} = (\rho+r)\mathbf{I} + \hat{\mathbf{G}}^T(k+1)\hat{\mathbf{G}}(k+1)$, $\mathbf{d} = 2\rho\mathbf{u}(k-1) -$

$$2\mathbf{y}(k) + 2\mathbf{y}_d + 2\hat{\mathbf{G}}(k+1)\mathbf{u}(k-1), \quad \mathbf{C} = \begin{bmatrix} \mathbf{I} \\ -\mathbf{I} \\ \mathbf{I} \\ -\mathbf{I} \\ \hat{\mathbf{G}}(k+1) \\ -\hat{\mathbf{G}}(k+1) \end{bmatrix},$$

$$\mathbf{c} = \begin{bmatrix} \mathbf{u}_{\max} \\ -\mathbf{u}_{\min} \\ \Delta\mathbf{u}_{\max} + \mathbf{u}(k-1) \\ -\Delta\mathbf{u}_{\max} - \mathbf{u}(k-1) \\ \mathbf{y}_{\max} + \hat{\mathbf{G}}(k+1)\mathbf{u}(k-1) + \mathbf{y}(k) \\ -\mathbf{y}_{\max} - \hat{\mathbf{G}}(k+1)\mathbf{u}(k-1) - \mathbf{y}(k) \end{bmatrix}. \quad \text{Since Eq. (21)}$$

is a quadratic program subject to linear inequality constraints, there are tractable methods such as active set method and interior point method [15] to be applied to on-line solve (21).

2.3 Robust convergence of the direct adaptive controller

Theorem 2: For any initial condition $\hat{\mathbf{u}}(0)$, if the following condition as Eq. (22) is satisfied, the direct adaptive controller as Eq. (17) has convergence property.

$$\sigma[\hat{\mathbf{G}}(k+1) - \mathbf{G}] < \frac{2\rho}{\sqrt{\rho+r}} \quad (22)$$

where, $\sigma[\hat{\mathbf{G}}(k+1) - \mathbf{G}]$ denotes the maximum singular value of matrix $[\hat{\mathbf{G}}(k+1) - \mathbf{G}]$.

Proof sees Appendix C.

According to **Theorem 2**, the proposed direct adaptive controller has the robust convergence property. The convergence robustness improves for larger value of the regulation parameter ρ in the direct adaptive controller and is absent if no regulation is performed.

2.4 Static control error

If the embedding matrix is estimated precisely, that means $\hat{\mathbf{G}}(l+1) = \mathbf{G}$ and control converges that means $\Delta\mathbf{u} = 0$, then according to Eq. (13) and Eq. (18),

$$\frac{dJ}{d\mathbf{u}} = 0, \text{ and the optimal control input } \mathbf{u}^* \text{ satisfies}$$

$$\rho\mathbf{u}^* + \mathbf{G}^T\mathbf{G}\mathbf{u}^* + \mathbf{G}^T[\mathbf{y}(0) - \mathbf{y}_d] = 0 \quad (23)$$

Multiplying Eq. (23) by $\mathbf{G}$ from left and according to Eq. (18),

$$\mathbf{y}^* - \mathbf{y}_d = (\mathbf{I} + \rho^{-1}\mathbf{G}\mathbf{G}^T)^{-1}[\mathbf{y}(0) - \mathbf{y}_d] \quad (24)$$

For $\rho > 0$, $\mathbf{y}^* - \mathbf{y}_d$ denotes the static control error. Consider a singular decomposition of $\mathbf{G}$ as

$$\mathbf{G} = \mathbf{A}\mathbf{\Gamma}\mathbf{B} \quad (25)$$

where, $\mathbf{A}$ and $\mathbf{B}$ are orthogonal matrices, $\mathbf{\Gamma} = \text{diag}\{a_i\}_{i=1}^m$ is a diagonal matrix. Then the static control error in Eq. (24) can be rewritten as

$$\mathbf{y}^* - \mathbf{y}_d = \mathbf{A}\mathbf{P}\mathbf{Z} \quad (26)$$

$$\mathbf{P} = \text{diag}\left\{\frac{1}{1 + \alpha_i^2 / \rho}\right\}_{i=1}^m \quad (27)$$

$$\mathbf{Z} = \mathbf{A}^T[\mathbf{y}(0) - \mathbf{y}_d] = [z_1, \dots, z_m]^T \quad (28)$$

Since $\mathbf{A}$ is an orthogonal matrix, from Eq. (26) to Eq. (28),

$$\|\mathbf{y}^* - \mathbf{y}_d\|^2 = \sum_{i=1}^m \frac{l_i^2}{(1 + \alpha_i^2 / \rho)^2} \quad (29)$$

If $\alpha_i^2 \gg \rho$, then $\|\mathbf{y}^* - \mathbf{y}_d\| \rightarrow 0$.

According to Eq. (29), the proposed direct adaptive controller has the asymptotic tracking control target property. At the same time, according to Eq. (29), increasing ρ increases the steady-state control error for the convergence achieved. Therefore, regulation parameter ρ should be chosen by balancing the convergence robustness and static control error.

2.5 Estimation of embedding matrix

According to Eq. (16), there is

$$\mathbf{G}(k) = \begin{bmatrix} \mathbf{g}_1(k)^T \\ \mathbf{g}_2(k)^T \\ \vdots \\ \mathbf{g}_n(k)^T \end{bmatrix} = \begin{bmatrix} g_{11}(k) & g_{12}(k) & \cdots & g_{1m}(k) \\ g_{21}(k) & g_{22}(k) & \cdots & g_{2m}(k) \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1}(k) & g_{n2}(k) & \cdots & g_{nm}(k) \end{bmatrix} \quad (30)$$

The Recursive Least Square (RLS) [16] as from Eq. (31) to Eq. (33) is used to estimate the embedding matrix $\mathbf{G}(k)$ at the time instant k , with the known data $\{\mathbf{u}(0), \mathbf{y}(1), [\mathbf{u}(1), \mathbf{y}(2)], \dots, [\mathbf{u}(k-1), \mathbf{y}(k)]\}$.

$$\begin{aligned} \hat{\mathbf{g}}_i(k) &= \hat{\mathbf{g}}_i(k-1) + \mathbf{m}^*(k)\{\mathbf{y}_i(k) - \mathbf{y}_i(k-1) - \\ &\quad \hat{\mathbf{g}}_i(k-1)^T \Delta \mathbf{u}(k-2)\}, \\ (i &= 1, \dots, n) \end{aligned} \quad (31)$$

$$\mathbf{m}^*(k) = \frac{\mathbf{P}^*(k-1)\Delta \mathbf{u}(k-1)}{\beta_1 + \Delta \mathbf{u}(k-1)^T \mathbf{P}^*(k-1)\Delta \mathbf{u}(k-1)} \quad (32)$$

$$\mathbf{P}^*(k) = \frac{1}{\beta_1} [\mathbf{I} - \mathbf{m}^*(k)\Delta \mathbf{u}(k-1)^T] \mathbf{P}^*(k-1) \quad (33)$$

where, $\beta_1 > 0 \in \mathbf{R}$ is the forgetting factor and $\mathbf{I}$ is the unit matrix.

Since the measurement data include the noise, the noise is filtered before estimating the embedding matrix. According to the principle of multi-resolution analysis [17], choose the basis scale $\phi(k)$ and wavelet $\psi(k)$ properly, $\{\psi_{j,l}(k) = 2^{-j} \psi(2^{-j}k - l), j = j_0 + 1, \dots, j_m, l \in \mathbf{Z}\}$ and $\{\phi_{m,l} = 2^{-j_m} \phi(2^{-j_m}k - l), l \in \mathbf{Z}\}$ can form the orthogonal basis in $L^2(\mathbf{R})$ space, where j is called scale factor, l is called location factor. Then, any

$\mathbf{y}(t) \in L^2(\mathbf{R})$ can be expanded as,

$$\mathbf{y}(k) = \sum_{j=j_0+1}^{j_m} \sum_l d_y(j, l) \psi_{j,l}(k) + \sum_l c_y(j_m, l) \phi_{j_m,l}(k) \quad (34)$$

The wavelet coefficients $d_y(j, l)$, $j = j_0 + 1, \dots, j_m$ and the scale coefficient $c_y(j_m, l)$ can be calculated with the fast wavelet transform algorithm of Mallat [18]. Since the wavelet coefficients $d_y(j, l)$, $j = j_0 + 1, \dots, j_m$ represent the high frequency information usually the noise signal in the measurement data, then some wavelet coefficients can be deleted to filter the noise in the measurement data. Due to the orthogonality of $\{\psi_{j,l}(k) = 2^{-j} \psi(2^{-j}k - l), j = j_0 + 1, \dots, j_m, l \in \mathbf{Z}\}$ and $\{\phi_{m,l} = 2^{-j_m} \phi(2^{-j_m}k - l), l \in \mathbf{Z}\}$, then

$$\|\mathbf{y}\|^2 = \sum_{j=j_0+1}^{j_m} \sum_l d_y(j, l)^2 + \sum_l c_y(j_m, l)^2 \quad (35)$$

The deletion of the wavelet coefficients can be done according to the following approximation rule

$$\|\mathbf{y} - \mathbf{y}_F\| = \left[\sum_{d_y(j, l) \in \mathbf{S}_-} d_y(j, l)^2 \right]^{\frac{1}{2}} < \varepsilon \quad (36)$$

where, $\mathbf{y}_F$ is the filtered measurement data, $\mathbf{S}_-$ is the set of deleted wavelet coefficients and ε is the chosen approximation limit which can be determined according to operating region change [19]. $\mathbf{y}_F$ can be rebuilt as

$$\mathbf{y}_F(k) = \sum_j \sum_l \tilde{d}_y(j, l) \psi_{j,l}(k) + \sum_l c_y(j_m, l) \phi_{j_m,l}(k) \quad (37)$$

where, $\tilde{d}_y(j, l)$ are the kept wavelet coefficients after the deletion. According to the filtered data $\{\mathbf{u}(0), \mathbf{y}_F(1), [\mathbf{u}(1), \mathbf{y}_F(2)], \dots, [\mathbf{u}(k-1), \mathbf{y}_F(k)]\}$, the embedding matrix $\mathbf{G}(k)$ can be estimated with RLS.

2.6 Adaptive to input-output time delay

For complex industrial processes, there is always some input-output time delay. Therefore, the controller design should take the effect of input-output time delay into account. Considering the input-output time delay, the embedding property can be described as

$$\mathbf{y}(k) - \mathbf{y}(k-1) = q^{-\tau} \mathbf{G}(k)[\mathbf{u}(k-1) - \mathbf{u}(k-2)] \quad (38)$$

where, q^{-1} is the delay operator and τ is the input-output delay. Let

$$\mathbf{y}^*(k) - \mathbf{y}^*(k-1) = \mathbf{G}^*(k)[\mathbf{u}(k-1) - \mathbf{u}(k-2)] \quad (39)$$

where, $\mathbf{G}^*(k)$ is the estimation of $\mathbf{G}(k)$ with RLS. Define

$$\hat{\mathbf{y}}(k+1) = \mathbf{y}(k+1) + \mathbf{y}^*(k+1) - \mathbf{y}^*(k+1 - \tau^*) \quad (40)$$

where, τ^* is the estimation of τ . Eq. (40) can be looked as a kind of the Smith predictor [20]. The optimal control objective is redefined as

$$\begin{aligned} \mathbf{J}(\mathbf{u}(k), \hat{\mathbf{y}}(k+1)) &= \rho \mathbf{u}^T \mathbf{u} + r \Delta \mathbf{u}^T \Delta \mathbf{u} + \\ &\quad (\hat{\mathbf{y}} - \mathbf{y}_d)^T (\hat{\mathbf{y}} - \mathbf{y}_d) \rightarrow \min \end{aligned} \quad (41)$$

Assume the $G(k)$ and τ are estimated precisely, let $\frac{\partial J}{\partial u} = 0$ and after some transformations, then

$$\begin{aligned}\hat{u}(k) = & u(k-1) - [(\rho + r)I + G^{*T}(k+1)G^*(k+1)]^{-1} \cdot \\ & [\rho u(k-1) + G^{*T}(k+1)(y(k) + y^*(k) - \\ & y^*(k - \tau^*) - y_d)]\end{aligned}\quad (42)$$

Comparing Eq. (42) with Eq. (17), the input-output time delay can be compensated when $G(k)$ and τ are estimated precisely.

For the complex industrial process, not only the process dynamic could be changed, but also the input-output time delay could be changed. Therefore, the following steps are applied to adjust the direct adaptive controller to adapt the input-output time delay change.

Step 1: Calculate $\Delta y(k) = y(k) - y(k-1)$, $\Delta y^*(k) = y^*(k) - y^*(k-1)$, where $\Delta y^*(k)$ is calculated with Eq. (42).

Step 2: Define the search range as $[\tau_1, \tau_2]$, $\tau_1 \leq \hat{\tau} \leq \tau_2$ and search step-length τ_{step} .

Step 3: τ^* is determined by

$$\tau^* = \min_{\tau_1 \leq \hat{\tau} \leq \tau_2} \|\Delta y(k) - \Delta y^*(k - \hat{\tau})\| \quad (43)$$

3 COMPARISON OF DIRECT ADAPTIVE CONTROL BASED ON GRADIENT ESTIMATION AND PID CONTROL

For simplification, the single-input single-output (SISO) system is considered. PID control (heurism based) is

$$u = k \left[e + \frac{1}{T_i} \int e(t) dt + T_d \frac{de}{dt} \right] \quad (44)$$

where, u , e , k , T_i , T_d are the manipulative variable, control error, proportion constant, integration time constant and difference time constant, respectively. The discrete increment form of PID can be represented as

$$\Delta u = k_1 \Delta e + k_2 e + k_3 \Delta^2 e \quad (45)$$

where, k_1 , k_2 , k_3 are constants. According to Eq. (14), direct adaptive control based on gradient estimation (optimization based) can be represented for SISO system as

$$u(k) = u(k-1) + \frac{\rho u(k-1) + g(k+1)e(k)}{\rho + r + g(k+1)^2} \quad (46)$$

If $\rho < 1$ is chosen, then

$$\Delta u = \frac{g}{r + g^2} e \quad (47)$$

Since g is the estimation of the gradient,

$$g = f(\Delta e, \Delta u) \quad (48)$$

where, function f can be acquired according to the gradient estimation algorithm. Therefore, the discrete

increment form of direct adaptive control based on gradient estimation can be represented as

$$\Delta u = f^*(e, \Delta e) \quad (49)$$

where, f^* is a nonlinear function. Therefore, comparing Eq. (45) and Eq. (49), the direct adaptive control based on gradient estimation actually is a nonlinear PI control. If f^* is differentiable, then

$$\begin{aligned}\Delta u = & f(e_0, \Delta e_0) + \left. \frac{\partial f}{\partial e} \right|_{e_0, \Delta e_0} \Delta e + \left. \frac{\partial f}{\partial \Delta e} \right|_{e_0, \Delta e_0} \Delta^2 e \\ \approx & k_1^* e + k_2^* \Delta e + k_3^* \Delta^2 e\end{aligned}\quad (50)$$

Comparing Eq. (45) and Eq. (50), PID control can be regarded as a linear case of the direct adaptive control based on gradient estimation.

Defining $f_1(g) \equiv \frac{g}{r + g^2}$, according to Eq. (47)

then

$$\Delta u = f_1(g)e \quad (51)$$

For linear system $f_1(g) = \text{constant}$, Δu is determined by control error e , just like the PI controller. For nonlinear system, Δu is determined by both $f_1(g)$ and control error e . According to Eq. (47), the increment of the control action is slowed down when the process changes quickly ($g \uparrow$, $f_1(g) \downarrow$ leads to $\Delta u \downarrow$) and the increment of the control action is speeded up when the process changes slowly ($g \downarrow$, $f_1(g) \uparrow$ leads to $\Delta u \uparrow$), therefore, the direct adaptive controller has the adaptive tracking ability to the operating point change of the nonlinear process, while the traditional PI controller is lack of such an ability.

Theorem 2 shows that the robustness bound deteriorates for larger r , but if r is too small, from Eq. (47) it can be seen that the control increment is determined by the gradient, when the gradient is also small, the control increment will be large, which makes the controller have a poor disturbance rejection property. Therefore, the regulation parameter r should be chosen by balancing the convergence robustness and disturbance rejection property.

4 INDUSTRIAL APPLICATION RESULTS

The industrial Himont spheripol license polypropylene process flow diagram is illustrated in Fig. 1. Hydrogen (H_2), propylene (C_3), catalyst (cat) and co-catalyst (co-cat) are fed to the prepolymerization reactor. The polymers from the prepolymerization together with propylene (C_3) are fed to the loop reactors, where the main polymerization reactions take place. The products of the loop reactors are separated in the flash tank, the gas are recycled as the feed to the prepolymerization reactor and the slurry is fed to the fluidized bed reactor and copolymerized with the feed ethylene (C_2) and propylene (C_3). The final product of polypropylene (PP) is gotten after the finishing of the copolymer from the fluid bed reactor. The polymerization reaction of polypropylene process is made up of following series of competing reactions,

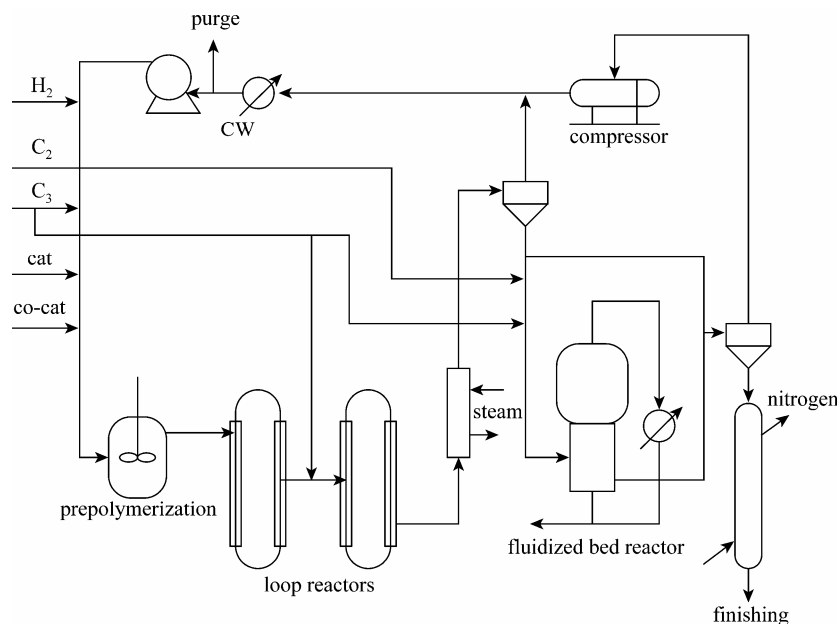
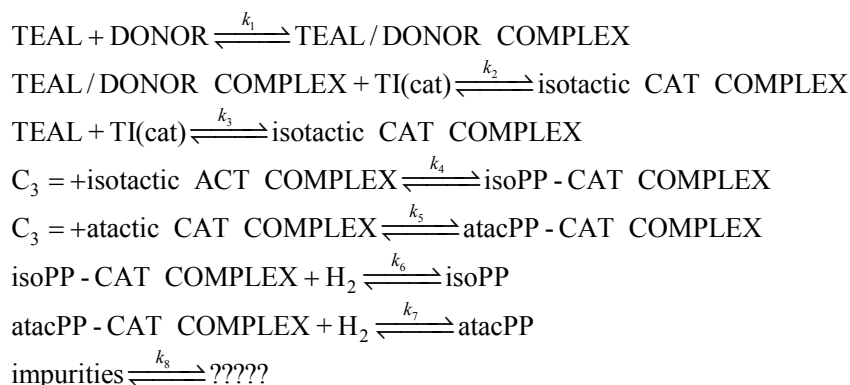
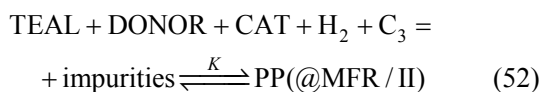


Figure 1 Himont spheripol license polypropylene process flow diagram



The general polymerization reaction of polypropylene can be described as



where, melt flow rate (MFR) and isotactic index (II) are two important PP grade index. MFR is a measure of flowability of molten polymer. It is directly correlated to chain length. A higher MFR is a thinner polymer. Isotactic index (II) is the percentage of the product that is isotactic PP. It is usually expressed as xylene solubles (XS) which is the percentage of polymer which will dissolve in xylene. Higher XS is needed for films and “living hinge” products. The reaction rates have the following relationships

$$K = \xi(T, k_1, k_2, \dots, k_8), \quad k_1, k_2, k_3 \succ k_4, k_5, k_6, k_7, k_8 \quad (53)$$

where, T is the reaction temperature and ξ is the unknown highly nonlinear function. From the above relationships, it can be seen that the reactor temperature is a direct measurement of the reaction kinetics, temperature fluctuations signify reaction kinetic fluctuations. The tighter the temperature control, the more

stable and safe the reaction is. Furthermore, better quality control is achieved. The reaction temperature control scheme is illustrated in Fig. 2, where TIC241 is loop reactor temperature, which is controlled by jacket cooling water inlet temperature TIC242. TIC242 is controlled by chilled water flow rate FIC271.

The hardware layout of an industrial Himont spheripol license PP plant is shown in Fig. 3, where the Distributed Control System (DCS) is the Honeywell Total Plant Solution (TPS) control system. The basic control loops are located in the App node. The proposed control method was configured as the reactor temperature advanced controller and connected to the TPS by the Ethernet with OPC (Object Linking and Embedding for Process Control). The reactor temperature advanced control is monitored by the operator and could be switched off if the performance of the advanced control was poor. Before the commissioning of the reactor temperature advanced control, the TIC242 control algorithm was robust PID [20] and the TIC241 control algorithm was profit loop [21], which actually is a single loop robust model predictive controller.

The control result of jacket water inlet temperature TIC242.PV with proposed method is illustrated in Fig. 4, where two-step lag embedding property was

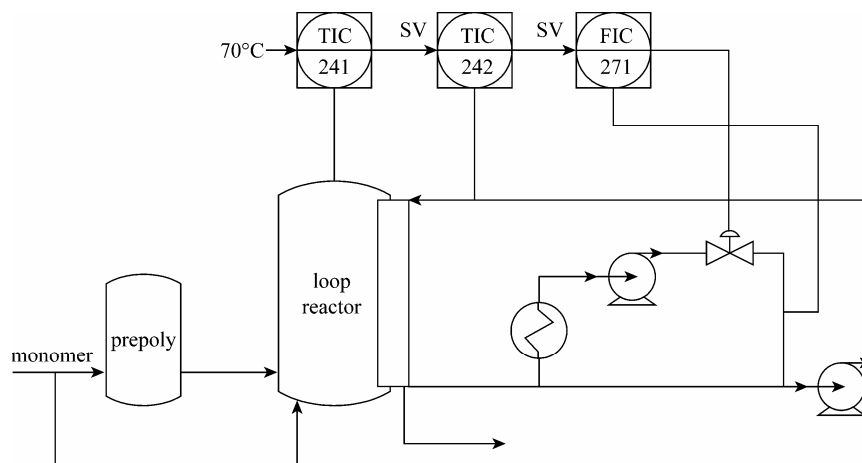


Figure 2 PP reaction temperature control scheme

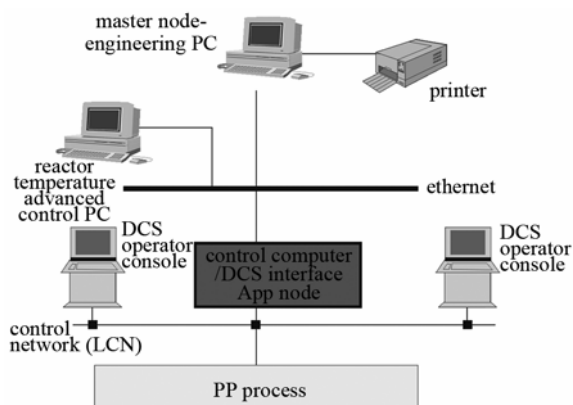


Figure 3 Hardware layout of an industrial Himont spheripol license PP plant

used, $\rho = 0.6$, $r = 0.4$, $P(0) = 1000$, $\hat{g}(0) = 0.5$, $\hat{g}(1) = 0.2$ and the input-output delay search range was [0.2 min, 0.4 min], search step-length was 0.05

min. The constraints for TIC242 control loop were configured as $|\text{TIC242.PV} - \text{TIC242.SV}| \leq 0.3^\circ\text{C}$, $708 \text{ kg}\cdot\text{h}^{-1} \leq \text{FIC271.SV} \leq 715 \text{ kg}\cdot\text{h}^{-1}$, $|\Delta \text{FIC271.SV}| \leq 2 \text{ kg}\cdot\text{h}^{-1}$. The Daubechies orthogonal wavelet transform [22] was applied to filter the measurement noise before estimating the embedding matrix, and ε was chosen as 5.57. For comparison, the control result with robust PID (RPID) is illustrated in Fig. 5. From Fig. 5, it can be seen that RPID controller can guarantee the control result under different operating point, but the adaptability of RPID is obviously less than the proposed method as illustrated in Fig. 4, even though the operating point change in Fig. 5 is much slower than that of in Fig. 4. Since TIC242 is the slave control loop in the reaction temperature control scheme, the fast tracking ability and adaptability are mostly desired. From this point of view, it can be seen that better control result is achieved with the proposed method.

The one-month control result of reaction temperature TIC241.PV with profit loop is illustrated in Fig. 6, where the predictive model was identified with

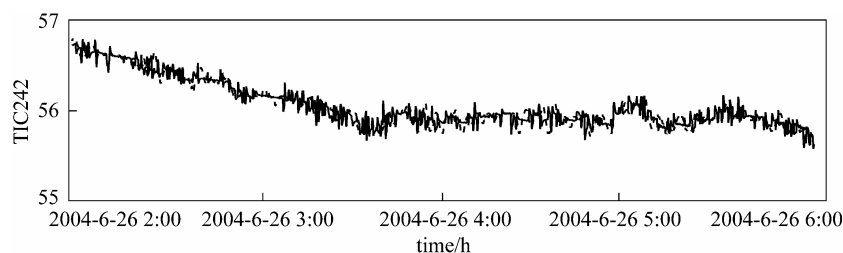


Figure 4 TIC242 control result with proposed method

— TIC242.SV; ---- TIC242.PV

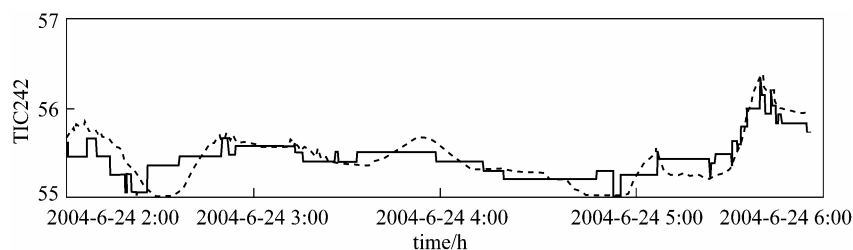


Figure 5 TIC242 control result with RPID

— TIC242.SV; ---- TIC242.PV

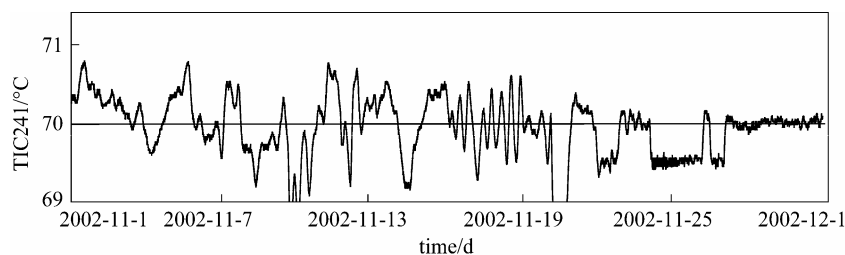


Figure 6 TIC241 control result with profit loop

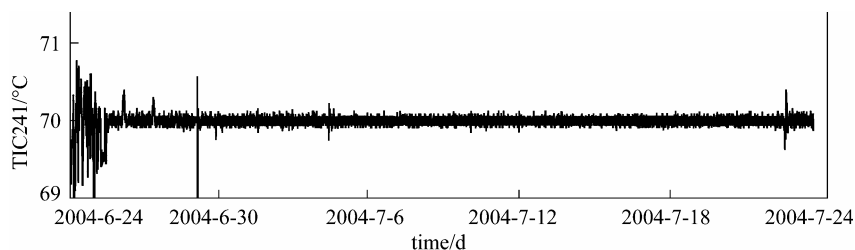


Figure 7 TIC241 control result with proposed method

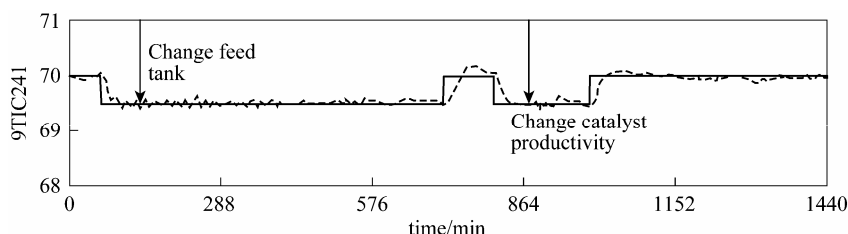


Figure 8 Robust tracking ability test with proposed method (from 2004-6-25 17:39 to 2004-6-26 17:39)
 — 9TIC241.SV; ---- 9TIC241.PV

profit stepper and profit identifier [23] based on the plant step test at nominal operating point as

$$\frac{y(s)}{u(s)} = \frac{0.273}{(s + 0.536)(s + 0.269)} e^{-0.349s} \quad (54)$$

According to Eq. (54), two-step lag embedding property was used, $\rho = 0.6$, $r = 0.4$, $P(0) = 1000$, $\hat{g}(0) = 1.89$, $\hat{g}(1) = 1.2$ and the input-output delay search range was [0.2 min, 0.4 min], search step was 0.05 min. The constraints for TIC241 control loop were configured as $|\text{TIC241.PV} - 70| \leq 0.5^\circ\text{C}$, $55^\circ\text{C} \leq \text{TIC242.SV} \leq 57^\circ\text{C}$, $|\Delta\text{TIC242.SV}| \leq 0.3^\circ\text{C}$. The Daubechies orthogonal wavelet transform was applied to filter the measurement noise before estimating the embedding matrix, and ε was chosen as 7.15. The one-month control result of reaction temperature TIC241.PV with proposed method is illustrated in Fig. 7. According to Fig. 7, the control result with proposed method is much better than that with profit loop after a short adaptation time. Since the PP process is a highly nonlinear process, a linear dynamic model, such as Eq. (54) identified in the neighborhood

of a nominal operating point cannot capture enough process characters which in turn reduces the performance of model predictive control even with the robust model predictive controller such as the profit loop.

To test the robust tracking ability of the proposed method, the process engineer changed the C_3 feed from tank 301 to tank 302 that could change C_3 feed features and also changed the productivity of catalyst; the control result with proposed method under the above disturbances is illustrated in Fig. 8. According to Fig. 8, the proposed method has strong robust tracking ability.

After one-year implementation of reaction temperature advanced control with proposed method, the benefit of advanced control was audited as Table 1. According to Table 1, the reaction temperature advanced control with proposed method improved the economic profit of PP plant obviously.

5 CONCLUSIONS

In this article, a systematic method of direct

Table 1 Benefit audit with proposed method based on 1-year data

	Reaction temperature standard control error/ $^\circ\text{C}$	Product rate / $\text{t}\cdot\text{h}^{-1}$	On-grade product percentage/%	Product grand transi- tion time/h
before Advanced Process Control (APC)	0.52	10.45 ± 0.98	61.47	8
after APC	0.049	11.7 ± 0.71	72.68	5.5

adaptive control based on gradient estimation is proposed. From the review of former related research, the above analysis and case study, the contribution of this article can be summarized as follows: (1) In process industry, to date, control systems for complex processes are implemented by experts on a case-by-case basis. The proposed directive adaptive controller, which is a generic controller based on the closed-loop input/output data, makes it possible that complex control system can be designed and implemented by average users with qualitative knowledge of the system only; (2) Direct adaptive controller can be designed based on the embedding property of dynamic system and proper choice of the controller parameters can make it have robust convergence and asymptotic unbiased tracking properties; (3) The application results of the proposed method to the industrial polypropylene process have verified its feasibility and effectiveness.

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APPENDIX

A. Proof of Theorem 1

Theorem 1: For dynamic system S , if $\mathbf{u}(k-2) = \mathbf{u}(k-1)$ leads to $\mathbf{y}(k-1) = \mathbf{y}(k-2)$, where $\mathbf{y}(k-1) = S[\mathbf{u}(k-2), \mathbf{Y}_{k-2}, \mathbf{U}_{k-3}]$, $\mathbf{y}(k) = S[\mathbf{u}(k-1), \mathbf{Y}_{k-1}, \mathbf{U}_{k-2}]$ then there always exists an embedding matrix $\mathbf{G}(k)$, satisfies

$$\mathbf{y}(k) - \mathbf{y}(k-1) = \mathbf{G}(k)[\mathbf{u}(k-1) - \mathbf{u}(k-2)] \quad (\text{a1})$$

Proof: If $\mathbf{u}(k-2) - \mathbf{u}(k-1) = 0$, then $\mathbf{y}(k-1) - \mathbf{y}(k-2) = 0$, therefore there exists arbitrary $\mathbf{G}_0 = \mathbf{G}(k)$, which satisfies Eq. (a1). If $\mathbf{u}(k-2) - \mathbf{u}(k-1) \neq 0$, let

$$\mathbf{G}(k) = \frac{[\mathbf{y}(k) - \mathbf{y}(k-1)][\mathbf{u}(k-1) - \mathbf{u}(k-2)]^T}{\|\mathbf{u}(k-1) - \mathbf{u}(k-2)\|^2} \quad (\text{a2})$$

where, $\|\cdot\|$ denotes the Euclid norm, then

$$\begin{aligned} & \mathbf{G}(k)[\mathbf{u}(k-1) - \mathbf{u}(k-2)] \\ &= \frac{[\mathbf{y}(k) - \mathbf{y}(k-1)][\mathbf{u}(k-1) - \mathbf{u}(k-2)]^T [\mathbf{u}(k-1) - \mathbf{u}(k-2)]}{\|\mathbf{u}(k-1) - \mathbf{u}(k-2)\|^2} \\ &= \mathbf{y}(k) - \mathbf{y}(k-1) \end{aligned} \quad (\text{a3})$$

B. Proof of Corollary 2

Corollary 2: If $\mathbf{g}(k)$ is an arbitrary vector, satisfies Eq. (6) and

$$\mathbf{g}(k) = \frac{\mathbf{u}(k-1) - \mathbf{u}(k-2)}{\|\mathbf{u}(k-1) - \mathbf{u}(k-2)\|^2} \{ \mathbf{y}(k) - \mathbf{y}(k-1) \} \quad (\text{b1})$$

where $\mathbf{u}(k-1) - \mathbf{u}(k-2) \neq 0$, $\|\cdot\|$ is the Euclid norm, then

$$\|\mathbf{g}(k)\|^2 = \min_{\mathbf{g}} \|\mathbf{g}\|^2 \quad (\text{b2})$$

Proof: Define Lagrangian function $f(\mathbf{g}, \lambda)$ as

$$f(\mathbf{g}, \lambda) = \|\mathbf{g}\|^2 + \lambda \{ [\mathbf{y}(k) - \mathbf{y}(k-1)] - \mathbf{g}^T [\mathbf{u}(k-1) - \mathbf{u}(k-2)] \} \quad (\text{b3})$$

where, $\lambda > 0$ is the Lagrangian multiplier.

$$\frac{\partial f}{\partial \mathbf{g}} = 2\mathbf{g} - \lambda [\mathbf{u}(k-1) - \mathbf{u}(k-2)] \quad (\text{b4})$$

$$\frac{\partial f}{\partial \lambda} = [\mathbf{y}(k) - \mathbf{y}(k-1)] - \mathbf{g}^T [\mathbf{u}(k-1) - \mathbf{u}(k-2)] \quad (\text{b5})$$

Let $\frac{\partial f}{\partial \mathbf{g}} = 0$, $\frac{\partial f}{\partial \lambda} = 0$, then

$$\lambda = \frac{2[\mathbf{y}(k) - \mathbf{y}(k-1)]}{\|\mathbf{u}(k-1) - \mathbf{u}(k-2)\|^2} \quad (\text{b6})$$

$$\mathbf{g} = \frac{\lambda}{2} [\mathbf{u}(k-1) - \mathbf{u}(k-2)] \quad (\text{b7})$$

Therefore,

$$\mathbf{g}(k) = \frac{\mathbf{u}(k-1) - \mathbf{u}(k-2)}{\|\mathbf{u}(k-1) - \mathbf{u}(k-2)\|^2} \{ \mathbf{y}(k) - \mathbf{y}(k-1) \} \quad (\text{b8})$$

Since there is only one minimum of $\|\mathbf{g}\|^2$, then the **Corollary 2** holds true.

C. Proof of Theorem 2

Theorem 2: For any initial condition $\hat{\mathbf{u}}(0)$, if the following condition as (c1) is satisfied, the direct adaptive controller as Eq. (17) has convergence property.

$$\sigma[\hat{\mathbf{G}}(k+1) - \mathbf{G}] < \frac{2\rho}{\sqrt{\rho+r}} \quad (\text{c1})$$

where, $\sigma[\hat{\mathbf{G}}(k+1) - \mathbf{G}]$ denotes the maximum singular value of matrix $[\hat{\mathbf{G}}(k+1) - \mathbf{G}]$.

Proof: By substituting Eq. (18) into Eq. (17) and after some transformations,

$$\hat{\mathbf{u}}(k) = \mathbf{D}(k) \left\{ r\mathbf{I} + \hat{\mathbf{G}}^T(k+1) [\hat{\mathbf{G}}^T(k+1) - \mathbf{G}] \right\} \mathbf{u}(k-1) - \mathbf{D}(k) \hat{\mathbf{G}}^T(k+1) [\mathbf{y}(0) - \mathbf{y}_d] \quad (\text{c2})$$

$$\mathbf{D}(k) = \left[(\rho+r)\mathbf{I} + \hat{\mathbf{G}}^T(k+1) \hat{\mathbf{G}}(k+1) \right]^{-1} \quad (\text{c3})$$

From Eq. (c3), it can be seen that the iterations converge if

$$\sigma \left\{ \mathbf{D}(k) \left[r\mathbf{I} + \hat{\mathbf{G}}^T(k+1) (\hat{\mathbf{G}}(k+1) - \mathbf{G}) \right] \right\} < 1 \quad (\text{c4})$$

The sufficient condition for Eq. (c4) to hold is

$$\sigma[\mathbf{D}(k)r\mathbf{I}] + \sigma \left\{ \mathbf{D}(k) \hat{\mathbf{G}}^T(k+1) [\hat{\mathbf{G}}(k+1) - \mathbf{G}] \right\} < 1 \quad (\text{c5})$$

By noting that $\sigma(\mathbf{D}(k)r\mathbf{I}) \leq r(\rho+r)^{-1}$, then the following condition of convergence is obtained

$$\sigma[\mathbf{D}(k) \hat{\mathbf{G}}^T(k+1)] \sigma[\hat{\mathbf{G}}(k+1) - \mathbf{G}] < \frac{\rho}{\rho+r} \quad (\text{c6})$$

Considering a singular value decomposition of $\hat{\mathbf{G}}(k+1)$ as

$$\hat{\mathbf{G}}(k+1) = \hat{\mathbf{A}} \hat{\mathbf{\Lambda}} \hat{\mathbf{B}} \quad (\text{c7})$$

where, $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$ are the orthogonal matrices, $\hat{\mathbf{\Lambda}} = \text{diag}\{\hat{\lambda}_i\}_{i=1}^m$ is a diagonal matrix. According to Eq. (c7),

$$\mathbf{D}(k) \hat{\mathbf{G}}^T(k+1) = \left[(\rho+r)\mathbf{I} + \hat{\mathbf{G}}^T(k+1) \hat{\mathbf{G}}^T(k+1) \right]^{-1} \cdot \hat{\mathbf{G}}^T(k+1) = \hat{\mathbf{B}}^T \hat{\mathbf{\Lambda}} \hat{\mathbf{A}}^T \quad (\text{c8})$$

$$\hat{\mathbf{\Lambda}} = \text{diag}\{\lambda_i\}_{i=1}^m \quad (\text{c9})$$

$$\lambda_i = \frac{\hat{\lambda}_i}{\rho+r+\hat{\lambda}_i^2}, \quad i=1, \dots, m \quad (\text{c10})$$

Since λ_i are the singular value of $\mathbf{D}(k) \hat{\mathbf{G}}^T(k+1)$, then according to Eq. (c6),

$$\sigma[\hat{\mathbf{G}}(k+1) - \mathbf{G}] (\max \lambda_i) < \frac{\rho}{\rho+r}, \quad i=1, \dots, m \quad (\text{c11})$$

Since for any real x , there is

$$\frac{x}{\rho+r+x^2} < \frac{1}{2\sqrt{\rho+r}} \quad (\text{c12})$$

According to Eq. (c10), there is

$$\max \lambda_i < \frac{1}{2\sqrt{\rho+r}}, \quad i=1, \dots, m \quad (\text{c13})$$

According to Eq. (c6), if Eq. (c14) holds true, then $\hat{\mathbf{u}}(k)$ converges.

$$\frac{1}{2\sqrt{\rho+r}} \sigma[\hat{\mathbf{G}}(k+1) - \mathbf{G}] < \frac{\rho}{\rho+r} \quad (\text{c14})$$

According to Eq. (c14), **Theorem 2** holds true.